HETS Online Journal
Volume 16, Issue 1: November 2025
ISSN 2693-9193

Shaping Problem-Solvers: The Role of Diverse Methods in Advancing Mathematical Confidence

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Abstract

This article investigates how exposure to diverse methods for solving quadratic equations influences the development of mathematical confidence and problem-solving skills among undergraduate students in community college algebra courses. Recognizing that students often enter with varying levels of prior knowledge and limited experience in flexible problem-solving, the instructional model introduced multiple strategies, including Grouping, Completing the Square, Trial and Error, the Quadratic Formula, and Slide-Divide-Bottoms-Up. Student perceptions, preferences, and self-reported confidence were collected through surveys and analyzed to assess both procedural proficiency and conceptual understanding. Findings reveal that engaging with multiple solution methods enhances students' confidence, encourages strategic thinking, and promotes adaptability in selecting appropriate approaches for different problem types. The results also demonstrate that students develop greater mathematical maturity, characterized by reflective reasoning, method comparison, and persistence in problem-solving. These insights suggest that integrating diverse methods into algebra instruction fosters a mindset of flexible and independent problem-solving, providing a pathway for students to approach complex mathematical challenges with confidence and resilience.

Keywords: Mathematical confidence, problem-solving strategies, algebra instruction, mathematical maturity, community college students, instructional methods

Resumen

Este artículo investiga cómo la exposición a diversos métodos para resolver ecuaciones cuadráticas influye en el desarrollo de la confianza matemática y las habilidades de resolución de problemas entre estudiantes universitarios de cursos de álgebra en colegios comunitarios. Reconociendo que los estudiantes a menudo ingresan con niveles variados de conocimiento previo y con experiencia limitada en la resolución flexible de problemas, el modelo de instrucción introdujo múltiples estrategias, incluidas Agrupación, Completar Cuadrado, el Ensayo y Error, la Fórmula Cuadrática y el método *Slide-Divide-Bottoms-Up*. Se recopilaron percepciones, preferencias y niveles de confianza auto informados de los estudiantes a través de encuestas y se analizaron para evaluar tanto la competencia procedimental como la comprensión conceptual. Los hallazgos revelan que interactuar con múltiples métodos de solución mejora la confianza de los estudiantes, fomenta el pensamiento estratégico y promueve la adaptabilidad al seleccionar enfoques apropiados para distintos tipos de problemas. Los resultados también demuestran que los estudiantes desarrollan una mayor madurez matemática, caracterizada por el razonamiento reflexivo, la comparación de métodos y la persistencia en la resolución de problemas. Estos hallazgos sugieren que la integración de diversos métodos en la enseñanza del álgebra fomenta una mentalidad de resolución flexible e independiente, proporcionando a los estudiantes una vía para abordar desafíos matemáticos complejos con confianza y resiliencia.

Palabras clave: confianza matemática, estrategias de resolución de problemas, enseñanza del álgebra, madurez matemática, estudiantes de colegios comunitarios, métodos de instrucción

Introduction

Algebra in community college classrooms tests students' ability to think, reason, and make decisions, not just apply formulas. Many students enter with limited experience in strategic problem-solving, often relying on rote methods. This study examines how exposure to multiple strategies for solving quadratic equations—including factoring, completing the square, the quadratic formula, and the Slide-and-Divide method—can enhance confidence, procedural skills, and conceptual understanding.

Despite the recognized importance of quadratic equations, little research has explored how exposure to multiple solution strategies shapes students into confident problem-solvers. Students often master procedures without developing flexible reasoning or self-assurance. This study investigates how multiple methods instruction can cultivate both competence and confidence.

Research Questions

1. How does exposure to multiple problem-solving methods influence students' development of mathematical confidence in algebra?
2. Which problem-solving strategies do students prefer when solving algebraic problems, and what does their choice reveal about their mathematical reasoning and maturity?
3. To what extent does engaging with various solution approaches improve students' procedural skills, conceptual understanding, and strategic thinking in algebra?

Literature Review

Quadratic equations are a central topic in algebra but challenging for students, often due to limited exposure to diverse problem-solving strategies (Erbas, 2015; Kotsopoulos, 2007; Zakaria & Maat, 2010). Research highlights the importance of addressing both procedural and conceptual dimensions.

Studies on factoring trinomials show that time and sequence affect perceived usefulness (Bass, 2021), while expanding factoring techniques sustains motivation (Bosse & Nandakumar, 2005). Early innovations include Dolorosa (1956) and Herrick (1907), and recent approaches include the Slip-and-Slide (Donnell, 2012) and Slide-Divide-Bottoms-Up methods (Chow, 2016; Hawthorn, 2015). Broadening students' repertoires enhances accessibility and engagement.

Difficulties extend beyond method selection to conceptual understanding. Students face challenges in formulating and solving quadratics (Erbas, 2015), abstraction (Kotsopoulos, 2007), and systematic errors (Zakaria & Maat, 2010). Cross-cultural studies reveal differences in persistence and accuracy (Vaiyavutjamai et al., 2005). These findings underscore the need for both procedural fluency and reflective reasoning.

Quadratics also develop what Steen (1988) called the "science of patterns" and what Tall (2004) described as the "three worlds of mathematics": embodied, symbolic, and formal. Tall, de Lima, and Healy (2014) illustrate how students' progress toward formal reasoning. Reflective problem-solving and persistence signal mathematical maturity (Faulkner, 2018; Spangler, 1992), supporting instructional approaches that emphasize method comparison and adaptability.

Quadratic equations provide a rich context for cultivating mathematical maturity. Exposure to multiple strategies—including factoring, completing the square, the quadratic formula, and heuristic methods—enhances confidence, motivation, and adaptability (Bass, 2021;

Donnell, 2012; Bosse & Nandakumar, 2005). Error analyses and cognitive studies caution that procedural fluency should be paired with conceptual understanding. Integrating these insights supports instruction that fosters confidence, persistence, and flexible problem-solving.

Methodology

Research Design

This study used a quantitative design supported by survey (Appendix A) to examine how solving quadratic affects students' mathematical maturity and confidence. It also explored students' perceptions of multiple problem-solving strategies and their impact on reasoning, reflection, and strategic decision-making.

Participants and Setting

The study included 43 undergraduate students in a community college algebra course, representing diverse academic backgrounds. Participation was voluntary, with informed consent obtained. Students learned multiple methods for solving quadratic equations: factoring by grouping, completing the square, trial-and-error, the quadratic formula, and Slide-Divide-Bottoms-Up. Instruction emphasized procedural fluency, conceptual understanding, and strategic choice.

Data Collection and Analysis

Data were collected via a structured questionnaire with Likert-scale, multiple-choice, and open-ended items. Quantitative responses were summarized with descriptive statistics, frequencies, and percentages. Open-ended responses were thematically coded to identify insights into problem-solving confidence, mathematical maturity, and instructional impact.

Teaching Implementation

Students explored four main methods: Grouping, Completing the Square, Trial-and-Error, and Slide-Divide-Bottoms-Up, comparing methods, selecting effective approaches, and reflecting and reasoning.

- **Completing the Square**: Emphasizes conceptual understanding and the relationship between algebraic structure and solution strategy (Bosse & Nandakumar, 2005).
- **Grouping Method**: Reinforced pattern recognition and structural reasoning (Dolorosa, 1956; Donnell, 2012).
- **Trial-and-Error Method**: Encouraged persistence and strategic experimentation (Bass, 2021).
- **Slide-Divide-Bottoms-Up Method**: Promoted efficiency and flexibility, though initially less intuitive (Chow, 2016b; Hawthorn, 2015; Herrick, 1907).

Students actively compared strategies, justified choices, and reported increased confidence in solving quadratic problems.

Results

This section presents the outcomes of a classroom-based study in which students explored multiple methods for solving quadratic equations. The analysis focuses on how these approaches impacted students' understanding, confidence, and mathematical maturity, based on survey responses and observed engagement.

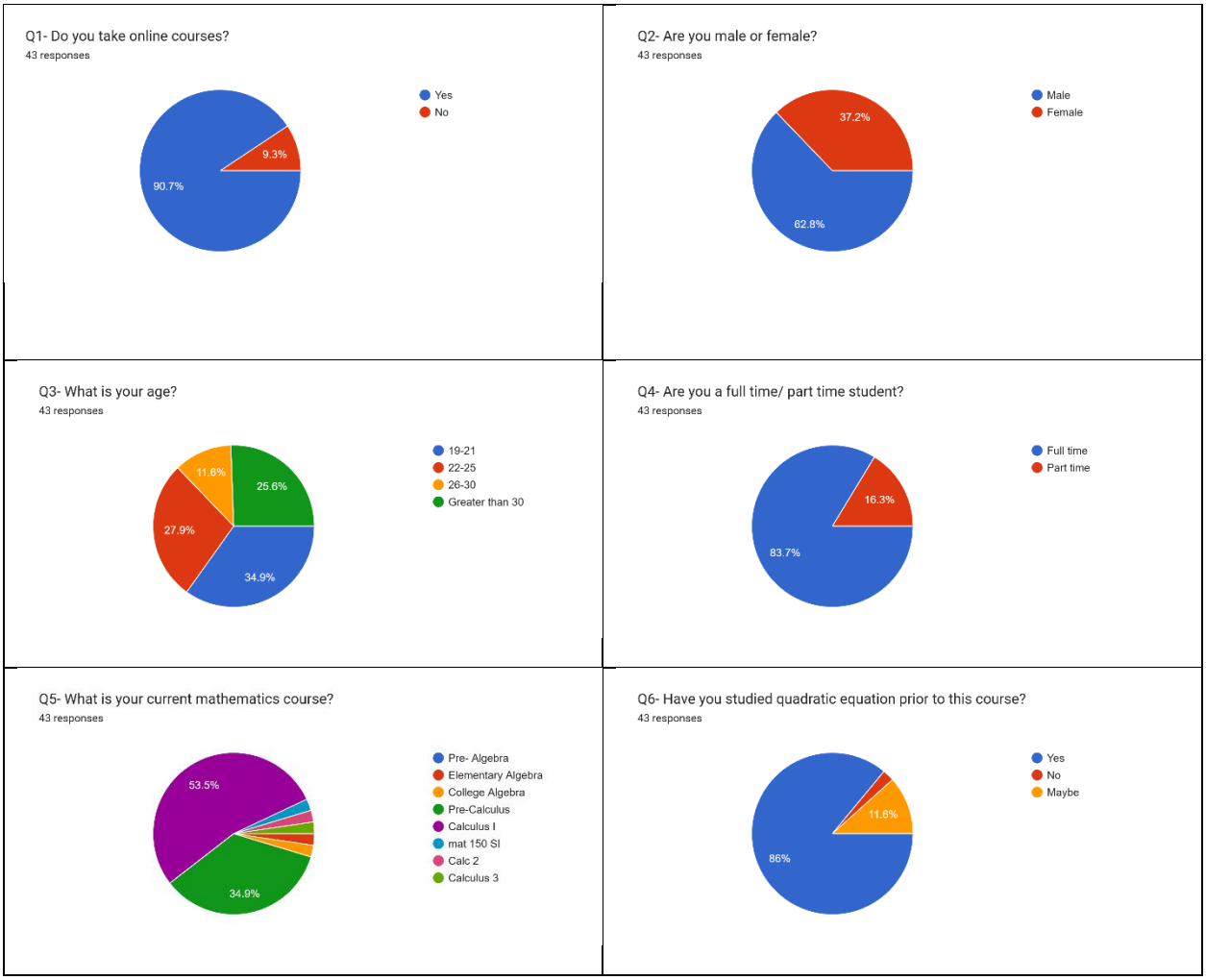
Background Information

Survey items Q1–Q6 gathered demographic and academic background data from the 43 student participants. As shown in Figure 1, most respondents (62.8%) identified as female and 37.2% as male. The age distribution reflected both traditional and nontraditional learners, with

34.9% over 30 years old and 27.9% between 26 and 30. Most students (83.7%) were enrolled full-time, while 90.7% reported taking at least one online course, indicating strong familiarity with digital learning environments. Regarding mathematics placement, 34.9% were enrolled in Pre-Calculus and 53.5% in Calculus I, suggesting a mix of intermediate and advanced backgrounds. Additionally, 86% reported prior exposure to quadratic equations, ensuring baseline familiarity with the study topic.

Figure 1

Demographic and Academic Background of Participants



Perception of Instructional Methods

Students' perceptions of the instructional methods introduced for solving quadratic equations are summarized in Table 1. The majority (69.1%) agreed that learning multiple methods enhanced their understanding, indicating strong support for a diversified instructional approach. The Grouping method was particularly valued, with 67.4% of students reporting that it clarified the process of factoring. The Trial-and-Error approach received moderate support (53.6%), though a substantial proportion of students remained neutral, suggesting that its perceived usefulness varied. Completing the Square was recognized as useful by 60.4% of students, despite 18.6% finding it challenging. The Slide-Divide-Bottoms-Up method showed promise, with 48.8% finding it effective and 46.5% neutral, likely reflecting initial unfamiliarity with this approach.

Table 1

Perception of Instructional Methods

Method	% Positive	% Neutral	% Negative	Notes
Learning multiple methods improved understanding	69.1%	23.8%	7.1%	Strong support
Grouping method clarified factoring	67.4%	27.9%	4.7%	Highly effective
Trial-and-Error	53.6%	39%	7.3%	Moderate usefulness
Completing the Square	60.4%	11.6%	18.6%	Useful but challenging
Slide-Divide-Bottoms-Up	48.8%	46.5%	4.7%	Promising, initial unfamiliarity

Mathematical Maturity and Problem-Solving Confidence

Exposure to multiple methods for solving quadratic equations was associated with significant gains in students' mathematical maturity and problem-solving confidence, as shown

in Table 2. A large majority (72.1%) reported increased confidence in solving quadratic equations independently, indicating enhanced self-efficacy. Most students (73.8%) expressed comfort with trying new approaches, demonstrating adaptability and a willingness to engage in strategic thinking. Additionally, 69.7% felt better prepared for more complex mathematical problems, suggesting that the experience with quadratics served as a strong foundation for future learning.

Table 2

Mathematical Maturity and Problem-Solving Confidence

Item	% Positive	% Neutral	% Negative	Key Insight
Confidence solving independently	72.1%	23.3%	4.6%	Increased self-efficacy
Comfortable trying new approaches	73.8%	21.4%	4.8%	Demonstrates adaptability
Prepared for complex problems.	69.7%	20.9%	9.3%	Quadratics as a foundation

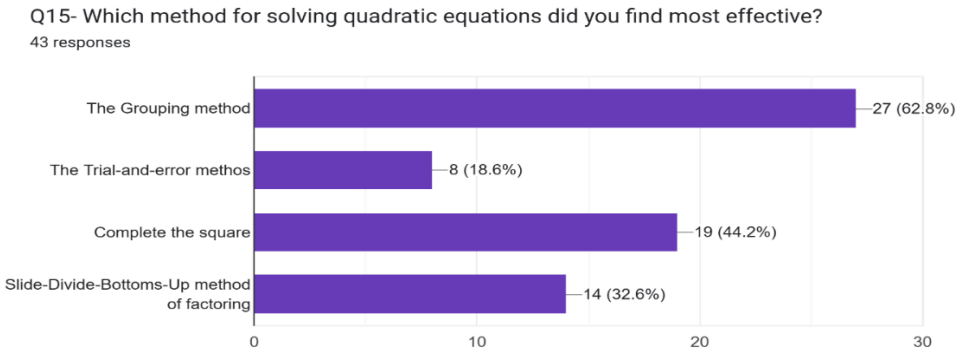
Preferences and Feedback

Student preferences for different problem-solving methods are illustrated in Figure 2. The Grouping method was favored by 62.8% of students for its clarity and organization, making it the most preferred approach. Completing the Square appealed to 44.2% of students, who cited its conceptual depth. The Slide-Divide-Bottoms-Up method attracted moderate interest (32.6%), while Trial-and-Error was least preferred (18.6%). Open-ended feedback reinforced these trends: students described Grouping as easy to remember, Slide-Divide-Bottoms-Up as helpful for complex trinomials and Completing the Square as deepening their understanding. Reported challenges included memory issues, difficulty selecting the most appropriate method, and arithmetic errors, particularly among returning students. Overall, exposure to multiple methods

promoted strategic thinking, method comparison, and confidence, with students recommending additional practice, step-by-step examples, and real-world applications.

Figure 2

Preferences



The results indicate that introducing multiple methods for solving quadratic equations not only improved students' procedural understanding but also fostered greater confidence, adaptability, and mathematical maturity. The Grouping method emerged as the most effective and preferred approach, while exposure to a variety of strategies encouraged students to think strategically and compare methods. These findings suggest that diversified instruction in quadratic equations can support both skill development and positive attitudes toward mathematics.

Lack of Mathematical Maturity

Over several semesters, we observed that many students hesitated to engage with advanced algebra problems, often due to fixed misconceptions or over-reliance on one familiar method. This resistance signaled a lack of mathematical maturity, particularly the flexibility and confidence needed to explore new strategies. Faulkner (2018) emphasizes that mathematical maturity is cultivated through exposure to unfamiliar problems that challenge students' reasoning

and require persistence. In our experience, once students began applying alternative methods in class such as grouping or completing the square many became more willing to take risks and explore challenging problems. This shift suggests that introducing new techniques can act as a catalyst for developing deeper mathematical thinking.

Research Question 1: How does exposure to multiple problem-solving methods influence students' development of mathematical confidence in algebra?

Exposure to a variety of problem-solving methods in algebra has a significant positive impact on students' mathematical confidence. In our data, approximately 72% of students reported feeling more capable of independently solving quadratic problems, 74% felt comfortable trying alternative strategies, and 70% felt prepared to tackle more complex problems. These findings are strongly supported by the literature, which consistently links the use of multiple solution strategies to increased self-efficacy and confidence in mathematics.

Research Question 2: Which problem-solving strategies do students prefer when solving algebraic problems, and what does their choice reveal about their mathematical reasoning and maturity?

Students most preferred the Grouping method (62.8%) for its clarity, followed by Completing the Square (44.2%) for conceptual understanding, and Slide-Divide-Bottoms-Up (32.6%) for efficiency. Trial-and-Error was least preferred (18.6%). These preferences suggest that students selecting conceptually driven methods demonstrate higher mathematical maturity, while those favoring procedural strategies prioritize efficiency and pattern recognition.

Research Question 3: To what extent does engaging with various solution approaches improve students' procedural skills, conceptual understanding, and strategic thinking in algebra?

Engaging with multiple methods enhanced procedural skills, with over 70% reporting improved accuracy and efficiency. Students also demonstrated stronger conceptual understanding and more strategic approaches to problem-solving. Reflections showed development in metacognitive skills such as planning, evaluating, and adjusting strategies, highlighting growth in maturity, reflective mathematical reasoning. For example, one student noted, *"I now try more than one method if I get stuck… I feel more confident and organized."* These reflections indicate the development of metacognitive skills, including planning, evaluating, and adjusting strategies—hallmarks of mature mathematical thinking.

Discussion

Exposure to multiple solution methods enhanced both procedural fluency and mathematical maturity. Students gained confidence, adaptability, and strategic awareness, key traits of effective problem-solvers. The Grouping method was particularly effective for easing cognitive load, while Completing the Square encouraged conceptual reasoning. Slide-Divide-Bottoms-Up offered efficiency but required further practice for comfort. Findings indicate that diversified instruction allows students to move beyond rote memorization, promoting reflective thinking, method evaluation, and problem-solving persistence. Learners developed flexibility in selecting appropriate strategies and applied reasoning rather than relying solely on formulas. Limitations include reliance on self-reported data and a cross-sectional design, which may not capture long-term retention or transfer to new problem types. Future research should incorporate longitudinal studies, performance-based assessments, and the exploration of additional algebraic contexts to measure sustained development of problem-solving skills and confidence.

Conclusion

This study examined how engaging students with diverse methods for solving quadratic equations contributes to shaping problem solvers and advancing mathematical confidence in introductory college algebra. Results indicate that exposure to multiple solution methods supports both conceptual understanding and procedural proficiency. Students reported increased confidence in choosing strategies, reflecting improved reasoning, self-monitoring, and strategic problem-solving. By comparing methods, recognizing patterns, and applying techniques thoughtfully, learners demonstrated growth in mathematical maturity.

These findings align directly with the goals outlined in the title. By providing opportunities to explore diverse approaches, students developed the habits and mindset of effective problem solvers. They became more willing to experiment with strategies, reflect on their reasoning, and make informed choices, which are essential components of confidence and adaptability in mathematics. While the study relied on self-reported data and a cross-sectional design, it offers evidence that integrating multiple problem-solving methods into algebra instruction can foster meaningful cognitive and affective growth. Future research should examine outcomes over time, performance-based measures, and sustained engagement with diverse methods to further enhance problem-solving abilities and confidence. Overall, emphasizing multiple solution strategies not only improves algebraic competence but cultivates resilience, reflective thinking, and strategic skills, equipping students for increasingly complex mathematical challenges.

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Appendix A

Student Survey: Shaping Problem-Solvers: The Role of Diverse Methods in Advancing Mathematical Confidence

Purpose:
This questionnaire is designed to gather student feedback and insights on how to learn and practice various methods, particularly the Slide-Divide-Bottoms-Up method for factoring—impacts your understanding and confidence in solving quadratic equations. Your responses will help evaluate how instructional strategies contribute to the development of mathematical maturity.

Instructions:
Please answer each question honestly. Your responses will remain confidential and will be used solely for educational research purposes. Your responses are important, particularly whether you use the Slide-Divide-Bottoms-Up method for factoring. Responses are confidential, no names, IDs or direct identifying information are asked for in this survey.
All submissions are voluntary. If you prefer to answer these questions on paper, then please e-mail me at **aamador@hostos.cuny.edu** and I will send you a copy. The estimated time to complete this survey is 10 minutes. Thanks for your time and determination.

Section A: Background Information

Q1- Do you take online courses?

- Yes
- No

Demographic information

Q2- Are you male or female?

- Yes
- No

Q3- What is your age?

- 19-21
- 22-25
- 26-30
- Greater than 30

Q4- Are you a full time/ part time student?

- Full time
- Part time

Q5- What is your current mathematics course?

- Pre-Algebra
- Elementary Algebra
- College Algebra
- Pre-Calculus
- Calculus I

- Other…

Q6- Have you studied quadratic equations prior to this course?

- Yes
- No
- Maybe

Section B: Perception of Instructional Methods

Q7- Learning several methods to solve quadratic equations helped me better understand the topic.

- Strongly disagree
- Disagree
- Neutral
- Agree
- Strongly agree

Q8- The grouping method made factoring easier for me to understand.

- Strongly disagree
- Disagree
- Neutral
- Agree
- Strongly agree

Q9- The Trial-and-error method was effective in helping me find roots of the quadratic equation.

- Strongly disagree
- Disagree
- Neutral
- Agree
- Strongly agree

Q10- Completing the square was challenging but a useful method.

- Strongly disagree
- Disagree
- Neutral
- Agree
- Strongly agree

Q11- The alternative method explained (Slide-Divide-Bottoms-Up method of factoring) helped me solve problems faster.

- Strongly disagree
- Disagree
- Neutral
- Agree
- Strongly agree

Section C: Mathematical Maturity and Problem-Solving Confidence

Q12- After learning different methods, I feel more confident solving quadratic equations on my own.

- Strongly disagree
- Disagree
- Neutral
- Agree
- Strongly agree

Q13- I feel comfortable trying different approaches to solve quadratic equations.

- Strongly disagree
- Disagree
- Neutral
- Agree
- Strongly agree

Q14- I now feel more prepared to tackle more complex algebra problems.

- Strongly disagree
- Disagree
- Neutral
- Agree
- Strongly agree

Section D: Preferences and Feedback

Q15- Which method for solving quadratic equations did you find most effective?

- The Grouping method
- The trial-and-error method
- Complete the square
- Slide-Divide-Bottoms-Up method of factoring

Q16- Please briefly explain why the method you selected was most effective for you:

Long answer text

Q17- What challenges did you face when solving quadratic equations during this unit?

Long answer text: _____

Q18- How did your approach to solving math problems change after this unit?

Long answer text: _____

Q19- What suggestions do you have to improve this unit for future students?

Long answer text: _____